

# Gravity-wave interferometers as probes of a low-energy effective quantum gravity

Giovanni AMELINO-CAMELIA<sup>1</sup>

Theory Division, CERN, CH-1211, Geneva, Switzerland

## ABSTRACT

The interferometry-based experimental tests of quantum properties of space-time which the author sketched out in a recent short Letter [Nature 398 (1999) 216] are here discussed in self-contained fashion. Besides providing detailed derivations of the results already announced in the previous Letter, some new results are also derived; in particular, the analysis is extended to a larger class of scenarios for space-time fuzziness and an absolute bound on the measurability of the amplitude of a gravity wave is obtained. It is argued that these studies could be helpful for the search of a theory describing a first stage of partial unification of Gravity and Quantum Mechanics.

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<sup>1</sup>*Marie Curie Fellow* of the European Union

# 1 INTRODUCTION AND SUMMARY

Perhaps the most fascinating questions confronting contemporary physics concern the search of the appropriate framework for the unified description of Gravity and Quantum Mechanics. This search for “Quantum Gravity” is proving very difficult [1], especially as a result of the scarce experimental information available on the interplay between Gravity and Quantum Mechanics. However, in recent years there has been a small (but nevertheless encouraging) number of new proposals [2, 3, 4, 5] of experiments probing the nature of the interplay between Gravity and Quantum Mechanics. At the same time the “COW-type” experiments, initiated with the celebrated experiment by Colella, Overhauser and Werner [6], have reached levels of sophistication [7] such that even gravitationally induced quantum phases due to local tides can be detected. In light of these developments there is now growing (although still understandably cautious) hope for data-driven insight into the structure of Quantum Gravity.

The primary objective of the present Article is the one of providing a careful discussion of the most recent addition to the (still far from numerous) family of Quantum Gravity experiments, which this author proposed in the short Letter in Ref. [8]. This most recent proposal probes in a rather direct way the properties of space-time, which is of course the most fundamental element of a Quantum Gravity, by exploiting the remarkable accuracy achievable with advanced modern interferometers, such as the ones used for searches of gravity waves.

While perhaps (especially in light of the gloom overall status of “Quantum Gravity phenomenology”) already sufficient interest in the experiment proposed in Ref. [8] could come from a pragmatic phenomenological viewpoint, in this Article I shall also relate the class of observations accessible to modern interferometers to a physical picture of the (necessarily small) way in which Quantum Gravity might affect phenomena probing space-time at distances significantly larger than the Planck length  $L_{\text{planck}} \sim 10^{-35}m$  (but significantly shorter than distance scales probed in ordinary particle-physics or gravity experiments). This physical picture is motivated by the huge gap between the minute Planck length and the distance scales probed in present-day particle-physics or gravitational experiments. The size of this gap provides motivation for exploring the possibility that on the way to Planck-length physics a few intermediate steps of partial unification of Gravity and Quantum Mechanics might be required before reaching full unification. Of course, as long as we are lacking direct experimental evidence to the contrary, it is also reasonable to work (as many distinguished colleagues do) on the hypothesis that Gravity and Quantum Mechanics should merge directly into a fully developed Quantum Gravity, but in the present Article (as in the previous papers [9, 10, 11]) I shall be concerned with the investigation of the properties that one could demand of a theory suitable for a first stage of partial unification of Gravity and Quantum Mechanics. In particular, I shall review the arguments presented in Refs. [10, 11] suggesting that the most significant implications of Quantum Gravity for low-energy (large-distance) physics might be associated to the structure of the non-trivial “Quantum Gravity vacuum”. A satisfactory picture of this Quantum Gravity vacuum is not available at present, and therefore we must generically characterize it as the appropriate new concept that in Quantum Gravity takes the place of the ordinary concept of “empty space”; however, it is plausible that some of the arguments by Wheeler, Hawking and others (see, *e.g.*, Refs. [12, 13] and references therein), which have attempted to develop an intuitive description of the Quantum Gravity vacuum, might have captured at least some of its actual properties.

Other possible elements for the search of a theory suitable for a first stage of partial unification of Gravity and Quantum Mechanics come from studies suggesting that this unification might require a novel relationship between “measuring apparatus” and “system”. My intuition on the nature of this new relationship is mostly based on work by Bergmann and Smith [14] and the observations I reported in Refs. [9, 11], which took as starting point an analysis by Salecker and Wigner [15].

The intuition emerging from these considerations on a novel relationship between measuring apparatus and system and by a Wheeler-Hawking picture of the Quantum Gravity vacuum are not sufficient for the full development of a new formalism describing the first stage of partial unification of Gravity and Quantum Mechanics, but they provide encouragement for the search of a formalism based on a mechanics not exactly of the type of ordinary Quantum Mechanics. Moreover, one can use this emerging intuition for rough estimates of certain candidate Quantum-Gravity effects. The estimates most relevant for the present Article are the ones concerning the space-time “fuzziness” which modern interferometers could investigate following Ref. [8].

A prediction of nearly all approaches to the unification of Gravity and Quantum Mechanics is that at very short distances the sharp classical concept of space-time should give way to a somewhat “fuzzy” (or “foamy”) picture (see, *e.g.*, Refs. [12, 13, 16, 17]), but it is usually very hard to characterize this fuzziness in physical operative terms. In Section 2 I provide an operative definition of fuzzy distance that has completely general applicability. My operative definition of fuzzy distance involves the use of interferometers, and the remarkable recent progress in the accuracy of these devices provides motivation for an analysis aimed at investigating the possible observable implications of Quantum Gravity for modern interferometers. In Section 3 I provide estimates for the quantum fluctuations that could affect distances if the above-mentioned intuition on the first stage of partial unification of Gravity and Quantum Mechanics is correct. I shall proceed with the attitude of searching for plausible (but admittedly “optimistic”) estimates of the relevant Quantum Gravity effects, and, although quantitative estimates will be derived, the true emphasis is on the qualitative aspects of the phenomena, since this type of information could be helpful to colleagues on the experimental side in establishing how to look for these phenomena. Some of the estimates I provide are motivated by studies of the measurability of distances in Quantum Gravity. A second group of estimates is based on elementary toy models of the stochastic processes that might characterize space-time fuzziness. The third and final group of estimates is motivated by arguments of “consistency” (in the sense discussed later) with recent proposals [4, 18, 19] of Quantum-Gravity induced deformation of the dispersion relation that characterizes the propagation of massless particles. All of these arguments indicate that a priority for interferometry-based tests of space-time fuzziness must be high sensitivity at low frequencies, and I hope this will be taken into account in planning future gravity-wave interferometers.

In Section 4 I shall observe (extending the related observations reported in Ref. [8]) that the remarkable sensitivity achieved by modern interferometers, especially the ones used to search for gravity waves [20, 21, 22, 23, 24], allows to set highly significant bounds on some of the fuzziness scenarios discussed in Section 2. Perhaps the most intuitive way to characterize the obtained bounds is given by the fact that we are now in a position to rule out a picture of fuzzy space-time such that minute Planck-length ( $10^{-35}m$ ) fluctuations would affect distances at a rate of one per each Planck time  $10^{-44}s$ . In Section 5 I derive a novel absolute bound on the measurability of the amplitude of a gravity wave. This measurability bound is obtained by combining a well-known “standard quantum limit,” which depends on the mass of the mirrors used by the gravity-wave interferometers, and a limitation on the mass of the mirrors that is imposed by gravitational effects. I find that this measurability bound is too weak to be tested with available or planned gravity-wave interferometers. Its significance mostly resides in the fact that it illustrates even more clearly than previous measurability analyses the fact that the unification of Gravity and Quantum Mechanics requires a new relationship between measuring apparatus and system. In Section 6 I discuss the aspects of certain existing Quantum Gravity approaches which are in one or another way related to the type of fuzzy space-times considered in Section 2. In Section 7 I discuss how the class of experiments proposed in Ref. [8] (and here analyzed in detail) complements other proposals of Quantum Gravity experiments. I also outline the general features that an experiment must have in order to uncover aspects of the interplay between Gravity and Quantum Mechanics. In

Section 8 I use the results discussed in Sections 2,3,4,5,6 to better define the idea of a theory appropriate for the description of a first stage of partial unification of Gravity and Quantum Mechanics. Closing remarks, also on the outlook for Quantum-Gravity phenomenology, are offered in Section 9.

## 2 OPERATIVE DEFINITION OF FUZZY DISTANCE

While nearly all approaches to the unification of Gravity and Quantum Mechanics appear to lead to a somewhat fuzzy picture of space-time, within the various formalisms it is often difficult to characterize physically this fuzziness. Rather than starting from formalism, I shall advocate an operative definition of fuzzy space-time.<sup>2</sup> More precisely for the time being I shall just consider the concept of fuzzy distance. I shall be guided by the expectation that at very short distances the sharp classical concept of distance should give way to a somewhat fuzzy distance. Since interferometers are ideally suited to monitor the distance between test masses, I choose as operative definition of Quantum-Gravity induced fuzziness one which is expressed in terms of Quantum-Gravity induced noise in the read-out of interferometers.

In order to articulate this proposal it will prove useful to briefly review some aspects of the physics of Michelson interferometers. These are schematically composed [20] of a (laser) light source, a beam splitter and two fully-reflecting mirrors placed at a distance  $L$  from the beam splitter in orthogonal directions. The light beam is decomposed by the beam splitter into a transmitted beam directed toward one of the mirrors and a reflected beam directed toward the other mirror; the beams are then reflected by the mirrors back toward the beam splitter, where [20] they are superposed<sup>3</sup>. The resulting interference pattern is extremely sensitive to changes in the positions of the mirrors relative to the beam splitter. The achievable sensitivity is so high that planned interferometers [22, 23] with arm lengths  $L$  of 3 or 4  $Km$  expect to detect gravity waves of amplitude  $h$  as low as  $3 \cdot 10^{-22}$  at frequencies of about  $100Hz$ . This roughly means that these modern gravity-wave interferometers should monitor the (relative) positions of their test masses (the beam splitter and the mirrors) with an accuracy [21] of order  $10^{-18}m$  and better.

In achieving this remarkable accuracy experimentalists must deal with classical-physics displacement noise sources (*e.g.*, thermal and seismic effects induce fluctuations in the relative positions of the test masses) and displacement noise sources associated to effects of ordinary Quantum Mechanics (as I shall mention again later the combined minimization of *photon shot noise* and *radiation pressure noise* leads to an irreducible noise source which has its root in ordinary Quantum Mechanics). The operative definition of fuzzy distance which I advocate characterizes the corresponding Quantum Gravity effects as an additional source of displacement noise. A theory in which the concept of distance is fundamentally fuzzy in this operative sense would be such that even in the idealized limit in which all classical-physics and ordinary Quantum-Mechanics noise sources are completely eliminated the read-out of an interferometer would still be noisy as a result of Quantum Gravity effects.

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<sup>2</sup>Once we have a physical definition of fuzzy space-time the analysis of the various Quantum Gravity formalisms could be aimed at providing predictions for this fuzziness. Of course, in order for the formalisms to provide such physical predictions it is necessary to equip them with at least some elements of a “measurement theory”.

<sup>3</sup>Although all modern interferometers rely on the technique of folded interferometer’s arms (the light beam bounces several times between the beam splitter and the mirrors before superposition), I shall just discuss the simpler “no-folding” conceptual setup. The readers familiar with the subject can easily realize that the observations here reported also apply to more realistic setups, although in some steps of the derivations the length  $L$  would have to be understood as the optical length (given by the actual length of the arms times the number of foldings).

Adopting this operative definition of fuzzy distance, interferometers are of course the natural tools for experimental tests of proposed space-time fuzziness scenarios. However, even the remarkable sensitivity estimate of order  $10^{-18}m$  given above is quite far from the Planck length  $\sim 10^{-35}m$ , and it might appear safe to assume that any scenario for space-time fuzziness would not observably affect the operation of even the most sophisticated modern interferometers. In spite of the intuition emerging from this preliminary considerations, in the next Sections 3 and 4 I shall show that some plausible (albeit somewhat speculative) fuzziness scenarios can be tested in a rather significant way by modern interferometers. The key observation is based on the fact that the physics of an interferometer involves other length scales besides the  $10^{-18}m$  length scale discussed above, and the combinations of length scales which characterize on the one hand the noise levels achievable by modern interferometers and on the other hand the Quantum-Gravity induced noise levels turn out to be comparable. In particular, a proper description of noise levels in an interferometer must provide the displacement sensitivity as a function of frequencies  $f$  (notice the additional length scale  $cf^{-1}$  obtained combining  $f$  with the speed-of-light constant  $c \sim 3 \cdot 10^8 m/s$ ), and similarly the “amount of fuzziness” predicted by certain space-time fuzziness scenarios turns out to be  $f$ -dependent. Within certain ranges of values of  $f$  one finds that the experimental limits are actually significant with respect to the theoretical predictions. Before providing this phenomenological analysis I shall use the next Section to discuss estimates of the type of noise levels that could be expected within certain space-time fuzziness scenarios.

### 3 SOME CANDIDATE FUZZY SPACE-TIMES

#### 3.1 Minimum-length noise

In many Quantum Gravity approaches there appears to be a length scale  $L_{min}$ , often identified with the string length ( $L_{string} \sim 10^{-34}m$ ) or the Planck length, which sets an absolute bound on the measurability of distances (a minimum uncertainty):

$$\delta D \geq L_{min} . \quad (1)$$

This property emerges in approaches based on canonical quantization of Einstein’s gravity when analyzing certain gedanken experiments (see, *e.g.*, Ref. [25, 26] and references therein). In Critical Superstring Theories, theories whose mechanics is still governed by the laws of ordinary Quantum Mechanics but with one-dimensional (rather than point-like) fundamental objects, a relation of type (1) follows from the stringy modification of Heisenberg’s uncertainty principle [27]

$$\delta x \delta p \geq 1 + L_{string}^2 \delta p^2 . \quad (2)$$

In fact, whereas Heisenberg’s uncertainty principle allows  $\delta x = 0$  (for  $\delta p \rightarrow \infty$ ), for all choices of  $\delta p$  the uncertainty relation (2) gives  $\delta x \geq L_{string}$ . The relation (2) is suggested by certain analyses of string scattering [27], but it might have to be modified when taking into account the non-perturbative solitonic structures of Superstring Theory known as Dirichlet branes [28]. In particular, evidence has been found [29] in support of the possibility that “Dirichlet particles” (Dirichlet 0 branes) could probe the structure of space-time down to scales shorter than the string length. In any case, all evidence available on Critical Superstring Theory is consistent with a relation of type (1), although it is probably safe to say that some more work is still needed to firmly establish the string-theory value of  $L_{min}$ .

Having clarified that a relation of type (1) is a rather common prediction of theoretical work on Quantum Gravity, let us then consider how such a relation could affect the noise

levels of an interferometer, *i.e.* let us consider the type of fuzziness (in the sense of the operative definition I advocated) which could be encoded in relation (1).

First let us observe that relation (1) does not necessarily encode any fuzziness; for example, relation (1) could simply emerge from a theory based on a lattice of points with spacing  $L_{min}$  and equipped with a measurement theory consistent with (1). The concept of distance in such a theory would not necessarily be affected by the type of stochastic processes that lead to noise in an interferometer.

However, it is also possible for relation (1) to encode the net effect of some underlying physical processes of the type one would qualify as quantum space-time fluctuations. These fluctuations, following work initiated by Wheeler and Hawking, are often visualized as involving geometry and topology fluctuations [12], virtual black holes [13], and other novel phenomena. A very intuitive description of the way in which the dynamics of matter distributions would be affected by this type of fuzziness of space-time is obtained by noticing certain similarities [30] between a thermal environment and the environment of quantum space-time fluctuations consistent with (1). This (however preliminary) network of intuitions suggests that (1) could be the result of fuzziness for distances  $D$  of the type associated to stochastic fluctuations with root-mean-square deviation  $\sigma_D$  given by

$$\sigma_D \sim L_{min} . \quad (3)$$

The associated displacement amplitude spectral density  $S_{min}(f)$  should roughly have a  $1/\sqrt{f}$  behaviour

$$S_{min}(f) \sim \frac{L_{min}}{\sqrt{f}} . \quad (4)$$

This can be justified using the observation that for a frequency-band limited from below only by the time of observation  $T_{obs}$  the relation between  $\sigma$  and  $S(f)$  is given by [31]

$$\sigma^2 = \int_{1/T_{obs}}^{f_{max}} [S(f)]^2 df . \quad (5)$$

Substituting the  $S_{min}(f)$  of Eq. (4) for the  $S(f)$  of Eq. (5) one obtains a  $\sigma$  that approximates the  $\sigma_D$  of Eq. (3) up to small (logarithmic)  $T_{obs}$ -dependent corrections. A more detailed description of the displacement amplitude spectral density associated to Eq. (3) can be found in Refs. [32, 33]. For the objectives of the present article the rough estimate (4) is sufficient since, if indeed  $L_{min} \sim L_{planck}$ , from (4) one obtains  $S_{min}(f) \sim 10^{-35} m/\sqrt{f}$ , which is still very far from the sensitivity of even the most advanced modern interferometers, and therefore we should not be concerned with corrections to Eq. (4).

### 3.2 Random-walk noise motivated by the analysis of a Salecker-Wigner gedanken experiment

The above argument relating the measurability bound (1) to fuzziness of type (3) can be used in general to relate any bound on the measurability of distances to an estimate of the possible stochastic quantum fluctuations affecting the operative definition of distances. In this Subsection 3.2 I shall consider a measurability bound that emerges when taking into account the quantum properties of devices. It is well understood (see, *e.g.*, Refs. [9, 11, 14, 34, 35, 36]) that the combination of the gravitational properties and the quantum properties of devices can have an important role in the analysis of the operative definition of gravitational observables. Since the analyses [25, 26, 27, 29] that led to the proposal of Eq. (3) only treated the devices in a completely idealized manner (assuming that one could ignore any contribution to the uncertainty in the measurement of  $D$  due to the gravitational

and quantum properties of devices), it is not surprising that analyses that took into account the gravitational and quantum properties of devices found more significant limitations to the measurability of distances.

Actually, by ignoring the way in which the gravitational properties and the quantum properties of devices combine in measurements of geometry-related physical properties of a system one misses some of the fundamental elements of novelty we should expect for the interplay of Gravity and Quantum Mechanics; in fact, one would be missing an element of novelty which is deeply associated to the Equivalence Principle. In measurements of physical properties which are not geometry-related one can safely resort to an idealized description of devices. For example, in the famous Bohr-Rosenfeld analysis [37] of the measurability of the electromagnetic field it was shown that the accuracy allowed by the formalism of ordinary Quantum Mechanics could only be achieved using idealized test particles with vanishing ratio between electric charge and inertial mass. Attempts to generalize the Bohr-Rosenfeld analysis to the study of gravitational fields (see, *e.g.*, Ref. [14]) are of course confronted with the fact that the ratio between gravitational “charge” (mass) and inertial mass is fixed by the Equivalence Principle. While ideal devices with vanishing ratio between electric charge and inertial mass can be considered at least in principle, devices with vanishing ratio between gravitational mass and inertial mass are not admissible in any (however formal) limit of the laws of gravitation. This observation provides one of the strongest elements in support of the idea [11] that the mechanics on which Quantum Gravity is based must not be exactly the one of ordinary Quantum Mechanics, since it should accommodate a somewhat different relationship between “system” and “measuring apparatus”. [In particular, the new mechanics should not rely on the idealized “measuring apparatus” which plays such a central role in the mechanics laws of ordinary Quantum Mechanics, see, *e.g.*, the “Copenhagen interpretation”.]

In trying to develop some intuition for the type of fuzziness that could affect the concept of distance in Quantum Gravity, it might be useful to consider the way in which the interplay between the gravitational and the quantum properties of devices affects the measurability of distances. In Refs. [9, 11] I have argued that a natural starting point for this type of analysis is provided by the procedure for the measurement of distances which was discussed in influential work by Salecker and Wigner [15]. These authors “measured” (in the “*gedanken*” sense) the distance  $D$  between two bodies by exchanging a light signal between them. The measurement procedure requires by *attaching*<sup>4</sup> a light-gun (*i.e.* a device capable of sending a light signal when triggered), a detector and a clock to one of the two bodies and *attaching* a mirror to the other body. By measuring the time  $T_{obs}$  (time of observation) needed by the light signal for a two-way journey between the bodies one also obtains a measurement of the distance  $D$ . For example, in Minkowski space and neglecting quantum effects one simply finds that  $D = cT_{obs}/2$ . Within this setup it is easy to realize that the interplay between the gravitational and the quantum properties of devices leads to an irreducible contribution to the uncertainty  $\delta D$ . In order to see this it is sufficient to consider the contribution to  $\delta D$  coming from the uncertainties that affect the motion of the center of mass of the system composed by the light-gun, the detector and the clock. Denoting with  $x^*$  and  $v^*$  the position and the velocity of the center of mass of this composite device relative to the position of the body to which it is *attached*, and assuming that the experimentalists prepare this device in a state characterised by uncertainties  $\delta x^*$  and  $\delta v^*$ , one easily finds [15, 11]

$$\delta D \geq \delta x^* + T_{obs} \delta v^* \geq \delta x^* + \left( \frac{1}{M_b} + \frac{1}{M_d} \right) \frac{\hbar T_{obs}}{2 \delta x^*} \geq \sqrt{\frac{\hbar T_{obs}}{2} \left( \frac{1}{M_b} + \frac{1}{M_d} \right)}, \quad (6)$$

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<sup>4</sup>Of course, for consistency with causality, in such contexts one assumes devices to be “attached non-rigidly,” and, in particular, the relative position and velocity of their centers of mass continue to satisfy the standard uncertainty relations of Quantum Mechanics.

where  $M_b$  is the mass of the body,  $M_d$  is the total mass of the device composed of the light-gun, the detector, and the clock, and the right-hand-side relation follows from observing that Heisenberg's *Uncertainty Principle* implies  $\delta x^* \delta v^* \geq (1/M_b + 1/M_d)\hbar/2$ . [N.B.: the *reduced mass*  $(1/M_b + 1/M_d)^{-1}$  is relevant for the relative motion.] Clearly, from (6) it follows that in order to eliminate the contribution to the uncertainty coming from the quantum properties of the devices it is necessary to take the formal "classical-device limit," *i.e.* the limit<sup>5</sup> of infinitely large  $M_d$ .

Up to this point I have not yet taken into account the gravitational properties of the devices and in fact the "classical-device limit" encountered above is fully consistent with the laws of ordinary Quantum Mechanics. From a physical/phenomenological and conceptual viewpoint it is well understood that the formalism of Quantum Mechanics is only appropriate for the description of the results of measurements performed by classical devices. It is therefore not surprising that the classical-device (infinite-mass) limit turned out to be required in order to reproduce the prediction  $\min \delta D = 0$  of ordinary Quantum Mechanics (which, as well known, allows  $\delta A = 0$  for any single observable  $A$ , since it only limits the combined measurability of pairs of conjugate observables).

If one also takes into account the gravitational properties of the devices, a conflict with ordinary Quantum Mechanics immediately arises because the classical-device (infinite-mass) limit is in principle inadmissible for measurements concerning gravitational effects.<sup>6</sup> As the devices get more and more massive they increasingly disturb the gravitational/geometrical observables, and well before reaching the infinite-mass limit the procedures for the measurement of gravitational observables cannot be meaningfully performed [9, 11, 35]. In the Salecker-Wigner measurement procedure the limit  $M_d \rightarrow \infty$  is not admissible when gravitational interactions are taken into account. At the very least the value of  $M_d$  is limited by the requirement that the apparatus should not turn into a black hole (which would not allow the exchange of signals required by the measurement procedure). These observations, which render unavoidable the  $\sqrt{T_{obs}}$ -dependence of Eq. (6), provide motivation for the possibility [9, 11] that in Quantum Gravity any measurement that monitors a distance  $D$  for a

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<sup>5</sup>A rigorous definition of a "classical device" is beyond the scope of this Article. However, it should be emphasized that the experimental setups being here considered require the devices to be accurately positioned during the time needed for the measurement, and therefore an ideal/classical device should be infinitely massive so that the experimentalists can prepare it in a state with  $\delta x \delta v \sim \hbar/M \sim 0$ . It is the fact that the infinite-mass limit is not accessible in a gravitational context that forces one to consider only "non-classical devices." This observation is not inconsistent with conventional analyses of decoherence for macroscopic systems; in fact, in appropriate environments, the behavior of a macroscopic device will still be "closer to classical" than the behavior of a microscopic device, although the limit in which a device has exactly classical behavior is no longer accessible.

<sup>6</sup>This conflict between the infinite-mass classical-device limit (which is implicit in the applications of the formalism of ordinary Quantum Mechanics to the description of the outcome of experiments) and the nature of gravitational interactions has not been addressed within any of the most popular Quantum Gravity approaches, including "Canonical/Loop Quantum Gravity" [38] and "Critical Superstring Theory" [28, 39]. In a sense somewhat similar to the one appropriate for Hawking's work on black holes [40], this "classical-device paradox" appears to provide an obstruction [11] for the use of the ordinary formalism of Quantum Mechanics for a description of Quantum Gravity.



time  $T_{obs}$  is affected by quantum fluctuations such that<sup>7</sup>

$$\delta D \geq \sqrt{L_{QG} c T_{obs}} , \quad (7)$$

where  $L_{QG}$  could in principle be an independent fundamental length scale (a length scale characterizing the nature of the novel Quantum-Gravity relationship between system and apparatus), but one is tempted to consider the possibility that  $L_{QG}$  be simply related to the Planck length. Interestingly, according to (7) the Salecker-Wigner measurement of a distance  $D$ , which requires a time  $2D/c$ , would be affected by an uncertainty of magnitude  $\sqrt{L_{QG} D}$ .

A  $\delta D$  that increases with  $T_{obs}$  (*e.g.* as in (7)) is not surprising for space-time fuzziness scenarios; in fact, the same phenomena that would lead to fuzziness are also expected to induce “information loss” [13] (the information stored in a quantum system degrades as  $T_{obs}$  increases). The argument based on the Salecker-Wigner setup provides motivation to explore the specific form  $\delta D \sim \sqrt{T_{obs}}$  of this  $T_{obs}$ -dependence.

Of course, the analyses reported above and in Ref. [9, 11] do not necessarily indicate that fuzziness of the type operatively defined in Section 2 should be responsible for the measurability bound (7). The intuitive/heuristic arguments I advocated can provide a (tentative) estimate of the measurability bound, but a full Quantum Gravity theory would be required in order to be able to determine which phenomena could be responsible for the bound. If one assumes that indeed fuzziness of the type operatively defined in Subsection 2 is responsible for the measurability bound (7) one is led to the possibility that a distance  $D$  would be affected by fundamental stochastic fluctuations with root-mean-square deviation  $\sigma_D$  given by

$$\sigma_D \sim \sqrt{L_{QG} c T_{obs}} . \quad (8)$$

From the type of  $T_{obs}$ -dependence of Eq. (8) it follows that the quantum fluctuations responsible for (8) should have displacement amplitude spectral density  $S(f)$  with the  $f^{-1}$  dependence<sup>8</sup> typical of “random walk noise” [31]:

$$S(f) = f^{-1} \sqrt{L_{QG} c} . \quad (9)$$

In fact, there is a general relation (which follows [31] from the general property (5)) between  $\sigma_D \sim \sqrt{T_{obs}}$  and  $S(f) \sim f^{-1}$ .

If indeed  $L_{QG} \sim L_{planck}$ , from (9) one obtains  $S(f) \sim f^{-1} \cdot (5 \cdot 10^{-14} m \sqrt{Hz})$ . As I shall discuss in detail later, by the standards of modern interferometers this noise level is quite

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<sup>7</sup>Note that Eq.(7) sets a minimum uncertainty which takes only into account the quantum and gravitational properties of the measuring apparatus. Of course, an even tighter bound might emerge when taking into account also the quantum and gravitational properties of the system under observation. However, according to the estimates provided in Refs. [25, 26] the contribution to the uncertainty coming from the system if of the type  $\delta D \geq L_{planck}$ , so that the total contribution (summing the system and the apparatus contributions) would be of the type  $\delta D \geq L_{planck} + \sqrt{L_{QG} c T_{obs}}$  which in nearly all contexts one can be concerned with (which would have  $c T_{obs} \gg L_{planck}^2$  can be approximated by completely neglecting the  $L_{planck}$  correction originating from the quantum and gravitational properties of the system.

<sup>8</sup>Of course, one expects that an  $f^{-1}$  dependence of the Quantum-Gravity induced  $S(f)$  could only be valid for frequencies  $f$  significantly smaller than the Planck frequency  $c/L_{planck}$  and significantly larger than the inverse of the time scale over which, even ignoring the gravitational field generated by the devices, the classical geometry of the space-time region where the experiment is performed manifests significant curvature effects.

significant, and therefore, before discussing other estimates of distance fuzziness, let us see whether the naive guess  $L_{QG} \sim L_{planck}$  can be justified within the argument used in arriving at (7). Since (7) was motivated from (6), and in going from (6) to (7) the scale  $L_{QG}$  was introduced to parametrize the minimum allowed value of  $1/M_b + 1/M_d$ , we could get some intuition for  $L_{QG}$  from trying to establish this minimum allowed value of  $1/M_b + 1/M_d$ . As mentioned, a conservative (possibly very conservative) estimate of this minimum value can be obtained by enforcing that  $M_b$  and  $M_d$  be at least sufficiently small to avoid black hole formation. In leading order (*e.g.*, assuming corresponding spherical symmetries) this amounts to the requirement that  $M_b < \hbar S_b / (c L_{planck}^2)$  and  $M_d < \hbar S_d / (c L_{planck}^2)$ , where the lengths  $S_b$  and  $S_d$  characterize the *sizes* of the regions of space where the matter distributions associated to  $M_b$  and  $M_d$  are localized. This observation implies

$$\frac{1}{M_b} + \frac{1}{M_d} > \frac{c L_{planck}^2}{\hbar} \left( \frac{1}{S_b} + \frac{1}{S_d} \right). \quad (10)$$

This suggests [9] that  $L_{QG} \sim \min[L_{planck}^2(1/S_b + 1/S_d)]$ :

$$\delta D \geq \min \sqrt{\left( \frac{1}{S_b} + \frac{1}{S_d} \right) \frac{L_{planck}^2 c T_{obs}}{2}}. \quad (11)$$

Of course, this estimate is very preliminary since a full Quantum Gravity would be needed here; in particular, the way in which black holes were handled in my argument might have missed important properties which would become clear only once we have the correct theory. However, it is nevertheless striking to observe that the naive guess  $L_{QG} \sim L_{planck}$  appears extremely far from the intuition emerging from this estimate; in fact,  $L_{QG} \sim L_{planck}$  would require that the maximum admissible value of  $S_d$  be of order  $L_{planck}$ . [I take  $S_b$  as fixed since it characterizes the size of the bodies whose distance is being measured, but of course the observer can choose the size  $S_d$  of the devices.] Since our analysis only holds for bodies and devices that can be treated as approximately rigid<sup>9</sup> and any non-rigidity would introduce additional contributions to the uncertainties, it is reasonable to assume that  $\max[S_d]$  be some small length (small enough that any non-rigidity would negligibly affect the measurement procedure), but the condition  $\max[S_d] \sim L_{planck}$  appears rather extreme. As I shall discuss in Section 4, already available experimental data rule out  $L_{QG} \sim L_{planck}$  in Eq. (9), and therefore if the  $f^{-1}$ -dependence of Eq. (9) is verified in the physical world (which is of course only one of the possibilities, and a rather speculative one)  $\max[S_d]$  must be somewhat larger than  $L_{planck}$ . As long as this type of analysis involves a  $\max[S_d]$  which is independent of  $\delta D$  one still finds  $\sqrt{T_{obs}}$ -dependence of  $\sigma_D$  (*i.e.*  $f^{-1}$ -dependence of  $S(f)$ ). If the correct Quantum Gravity is such that something like (11) holds but with  $\max[S_d]$  that depends on  $\delta D$ , one would have a different  $T_{obs}$ -dependence (and corresponding  $f$ -dependence), as I shall show in one example discussed in Subsection 3.6.

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<sup>9</sup>The fact that I have included only one contribution from the quantum properties of the devices, the one associated to the quantum properties of the motion of the center of mass, implicitly relies on the assumption that the devices and the bodies can be treated as approximately rigid. Any non-rigidity of the devices would of course introduce additional contributions to the uncertainty in the measurement of  $D$ . I shall further comment on the additional uncertainties that are introduced by the non-rigidity of devices in Section 5, where I consider some properties of the mirrors used in gravity-wave interferometry.

### 3.3 Random-walk noise from random-walk models of quantum space-time fluctuations

Since in this Article, like in Ref. [8], I am advocating a rather pragmatic phenomenological approach to Quantum Gravity, and taking into account the operative definition of fuzzy distance given in Section 2, it seems reasonable to consider the possibility that the properties of a distance  $D$  in a quantum space-time would involve a fluctuation of magnitude  $L_{\text{planck}} \sim 10^{-35}m$  over each time interval  $t_{\text{planck}} = L_{\text{planck}}/c \sim 10^{-44}s$ . The type of interferometer noise that would result from such a random-walk model of quantum space-time has the same qualitative structure as the noise I discussed in the previous Subsection motivated by the Salecker-Wigner measurement procedure. In fact, experiments monitoring the distance  $D$  between two bodies for a time  $T_{\text{obs}}$  (in the sense appropriate, *e.g.*, for a gravity-wave interferometer) would involve a total effect associated to quantum space-time amounting to  $n_{\text{obs}} \equiv T_{\text{obs}}/t_{\text{planck}}$  randomly directed fluctuations of magnitude  $L_{\text{planck}}$ . An elementary analysis allows to establish that in such a context the root-mean-square deviation  $\sigma_D$  would be proportional to  $\sqrt{T_{\text{obs}}}$ :

$$\sigma_D \sim \sqrt{L_{\text{planck}} c T_{\text{obs}}} . \quad (12)$$

We encounter again the  $\sqrt{T_{\text{obs}}}$ -dependence already considered in relation to the analysis of the Salecker-Wigner measurement procedure. Of course, this means that also for this random-walk models of quantum space-time the displacement amplitude spectral density has the characteristic  $f^{-1}$  behaviour. It also means that Eq. (12) as it stands predicts too much fuzziness. Therefore, if such a random-walk model of quantum space-time is verified in the physical world it must be that some of the simplifying assumptions made in deriving Eq. (12) were too naive. One possibility one might want to consider is the one in which the quantum properties of space-time are such that fluctuations of magnitude  $L_{\text{planck}}$  would occur with frequency somewhat lower than  $1/t_{\text{planck}}$ .

In closing this Subsection it seems worth adding a few comments on the stochastic processes here considered. In most physical contexts a series of random steps does not lead to  $\sqrt{T_{\text{obs}}}$  dependence of  $\sigma$  because often the context is such that through the fluctuation-dissipation theorem the source of  $\sqrt{T_{\text{obs}}}$  dependence gets tempered. The hypothesis explored in this Subsection, which can be partly motivated from the analysis of the Salecker-Wigner measurement procedure reported in the previous Subsection, is that the type of underlying dynamics of quantum space-time be such that the fluctuation-dissipation theorem be satisfied without spoiling the  $\sqrt{T_{\text{obs}}}$  dependence of  $\sigma$ . This is an intuition which apparently is shared by other authors; in fact, the study reported in Ref. [41] (which followed by a few months Ref. [8], but clearly was the result of completely independent work) also models some implication of quantum space-time (the ones that affect clocks) with stochastic processes whose underlying dynamics does not produce any dissipation and therefore the “fluctuation contribution” to the  $T_{\text{obs}}$  dependence remains unaffected, although the fluctuation-dissipation theorem is fully taken into account.

Since the mirrors of interferometers are basically extremities of a pendulum, another aspect that the reader might at first find counter-intuitive is that the  $\sqrt{T_{\text{obs}}}$  dependence of  $\sigma$ , although coming in with a very small prefactor, for extremely large  $T_{\text{obs}}$  would seem to give values of  $\sigma$  too large to be consistent with the structure of a pendulum. This is a misleading intuition which originates from the experience with ordinary (non-Quantum-Gravity) analyses of the pendulum. In fact, the dynamics of an ordinary pendulum has one extremity “fixed” to a very heavy and rigid body, while the other extremity is fixed to a much lighter body. The usual stochastic processes considered in the study of the pendulum affect the heavier body in a totally negligible way, while they have strong impact on the dynamics of the lighter body. A pendulum analyzed in the spirit of the present Subsection would be affected by stochastic processes which are of the same magnitude both for its heavier and its

lighter extremity. In particular in the directions orthogonal to the vertical axis the stochastic processes affect the position of the center of mass of the entire pendulum just as they would affect the position of the center of mass of any other body (the string that connects the two extremities of the pendulum would not affect the motion of its center of mass).

### 3.4 Random-walk noise motivated by linear deformation of dispersion relation

Both the analysis of the Salecker-Wigner measurement procedure and the analysis of simple-minded random-walk models of quantum space-time fluctuations have provided some encouragement for the study of interferometer noise of random-walk type. A third candidate Quantum Gravity effect that provides some encouragement for the random-walk noise scenario has emerged in the context of studies [4, 18, 19, 42, 43] of Quantum-Gravity induced deformation of the dispersion relation that characterizes the propagation of massless particles.

Deformed dispersion relations are not uncommon in the Quantum Gravity literature. For example, they emerge naturally in Quantum Gravity scenarios requiring a modification of Lorentz symmetry. Modifications of Lorentz symmetry could result from space-time discreteness, a possibility extensively investigated in the Quantum Gravity literature (see, *e.g.*, Ref. [44]), and it would also naturally result from an “active” Quantum-Gravity vacuum of the type advocated by Wheeler and Hawking [12, 13] (such a vacuum might physically label the space-time points).

While most Quantum-Gravity approaches will lead to deformed dispersion relations, the specific structure of the deformation can differ significantly from model to model. Assuming that the deformation admits a series expansion at small energies  $E$ , and parametrizing the deformation in terms of an energy<sup>10</sup> scale  $E_{QG}$  (a scale characterizing the onset of Quantum-Gravity dispersion effects, often identified with the Planck energy  $E_{planck} \sim 10^{19} GeV$ ), one would expect to be able to approximate the deformed dispersion relation at low energies according to

$$c^2 \mathbf{p}^2 \simeq E^2 \left[ 1 + \xi \left( \frac{E}{E_{QG}} \right)^\alpha \right] \quad (13)$$

where the power  $\alpha$  and the sign ambiguity  $\xi = \pm 1$  would be fixed in a given dynamical framework. For example, in some of the approaches based on dimensionful “ $\kappa$ ” quantum deformations of Poincaré symmetries [42, 43] one finds evidence of a dispersion relation for massless particles  $c^2 \mathbf{p}^2 = E_{QG}^2 \left[ 1 - e^{E/E_{QG}} \right]^2$ , and therefore  $\xi = \alpha = 1$ .

Scenarios (13) with  $\alpha = 1$  are in a sense consistent with random-walk noise. In fact, an experiment involving as a device (as a probe) a massless particle satisfying the dispersion relation (13) with  $\alpha = 1$  would be naturally affected by a device-induced uncertainty that grows with  $\sqrt{T_{obs}}$ . This is for example true in Quantum-Gravity scenarios in which the Hamiltonian equation of motion  $\dot{x}_i = \partial H / \partial p_i$  is still valid (at least approximately), where the deformed dispersion relation (13) leads to energy-dependent velocities for massless particles [4, 18, 42, 43]

$$v \simeq c \left[ 1 - \left( \frac{1 + \alpha}{2} \right) \xi \left( \frac{E}{E_{QG}} \right)^\alpha \right] , \quad (14)$$

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<sup>10</sup>I parametrize deformations of dispersion relations in terms of an energy scale  $E_{QG}$ , which is implicitly assumed to be rather close to  $E_{planck}$ , while I parametrize the proposals for measurability bounds with a length scale  $L_{QG}$ , which is implicitly assumed to be rather close to  $L_{planck}$ . This is somewhat redundant, since of course  $E_{planck} = \hbar c / L_{planck}$ , but it can help the reader in identifying the origin of a conjectured fuzziness scenario by simply looking at the type of parametrization that describes the stochastic processes.

and consequently the uncertainty in the position of the massless probe when a time  $T_{obs}$  has lapsed since the observer (experimentalist) set off the measurement procedure is given by

$$\delta x \simeq c \delta t + \delta v T_{obs} \simeq c \delta t + \frac{1 + \alpha}{2} \alpha \frac{E^{\alpha-1} \delta E}{E_{QG}^\alpha} c T_{obs} , \quad (15)$$

where  $\delta t$  is the quantum uncertainty in the time of emission of the probe,  $\delta v$  is the quantum uncertainty in the velocity of the probe,  $\delta E$  is the quantum uncertainty in the energy of the probe, and I used the relation between  $\delta v$  and  $\delta E$  that follows from (14). Since the quantum uncertainty in the time of emission of a particle and the quantum uncertainty in its energy are related<sup>11</sup> by  $\delta t \delta E \geq \hbar$ , Eq. (15) can be turned into an absolute bound on the uncertainty in the position of the massless probe when a time  $T_{obs}$  has lapsed since the observer set off the measurement procedure:

$$\delta x \geq c \frac{\hbar}{\delta E} + \frac{1 + \alpha}{2} \alpha \frac{E^{\alpha-1} \delta E}{E_{QG}^\alpha} T_{obs} \geq \sqrt{\left(\frac{\alpha + \alpha^2}{2}\right) \left(\frac{E}{E_{QG}}\right)^{\alpha-1} \frac{c^2 \hbar T_{obs}}{E_{QG}}} , \quad (16)$$

where I also used the fact that in principle the observer can prepare the probe in a state with desired  $\delta t$ , so it is legitimate to minimize the uncertainty with respect to the free choice of  $\delta t$ .

For  $\alpha = 1$  the  $E$ -dependence on the right-hand side of Eq. (16) disappears and one is led again (see Subsections 3.2 and 3.3) to a  $\delta x$  of the type  $(constant) \cdot \sqrt{T_{obs}}$ :

$$\delta x \geq \sqrt{\frac{c^2 \hbar T_{obs}}{E_{QG}}} . \quad (17)$$

When massless probes are used in the measurement of a distance  $D$ , as in the Salecker-Wigner measurement procedure, the uncertainty (17) in the position of the probe translates directly into an uncertainty on  $D$ :

$$\delta D \geq \sqrt{\frac{c^2 \hbar T_{obs}}{E_{QG}}} . \quad (18)$$

This was already observed in Refs. [10, 18, 43] which considered the implications of deformed dispersion relations (13) with  $\alpha = 1$  for the Salecker-Wigner measurement procedure.

Since deformed dispersion relations (13) with  $\alpha = 1$  have led us to the same measurability bound already encountered both in the analysis of the Salecker-Wigner measurement procedure and the analysis of simple-minded random-walk models of quantum space-time fluctuations, if we assume again that such measurability bounds emerge in a full Quantum Gravity as a result of corresponding quantum fluctuations (fuzziness), we are led once again to random-walk noise:

$$\sigma_D \sim \sqrt{\frac{c^2 \hbar T_{obs}}{E_{QG}}} . \quad (19)$$

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<sup>11</sup>It is well understood that the  $\delta t \delta E \geq \hbar$  relation is valid only in a weaker sense than, say, Heisenberg's Uncertainty Principle  $\delta x \delta p \geq \hbar$ . This has its roots in the fact that the time appearing in Quantum-Mechanics equations is just a parameter (not an operator), and in general there is no self-adjoint operator canonically conjugate to the total energy, if the energy spectrum is bounded from below [45, 41]. However, the  $\delta t \delta E \geq \hbar$  relation does relate  $\delta t$  intended as quantum uncertainty in the time of emission of a particle and  $\delta E$  intended as quantum uncertainty in the energy of that same particle.

### 3.5 Noise motivated by quadratic deformation of dispersion relation

In the preceding Subsection 3.4 I observed that Quantum-Gravity deformed dispersion relations (13) with  $\alpha = 1$  can also motivate random-walk noise  $\sigma_D \sim (\text{constant}) \cdot \sqrt{T_{obs}}$ . If we use the same line of reasoning that connects a measurability bound to a scenario for fuzziness when  $\alpha \neq 1$  we find  $\sigma_D \sim c(E/E_{QG}) \cdot \sqrt{T_{obs}}$ , where  $c(E/E_{QG})$  is a ( $\alpha$ -dependent) function of  $E/E_{QG}$ . However, in these cases with  $\alpha \neq 1$  clearly the connection between measurability bound and fuzzy-distance scenario cannot be too direct; in fact, the energy of the probe  $E$  which naturally plays a role in the context of the derivation of the measurability bound does not have a natural counter-part in the context of the conjectured fuzzy-distance scenario.

In order to preserve the conjectured connection between measurability bounds and fuzzy-distance scenarios one can be tempted to envision that if  $\alpha \neq 1$  the interferometer noise levels induced by space-time fuzziness might be of the type [see Eq. (16)]

$$\sigma_D \sim \sqrt{\left(\frac{\alpha + \alpha^2}{2}\right) \left(\frac{E^*}{E_{QG}}\right)^{\alpha-1} \frac{c^2 \hbar T_{obs}}{E_{QG}}}, \quad (20)$$

where  $E^*$  is some energy scale characterizing the physical context under consideration. [For example, at the intuitive level one might conjecture that  $E^*$  could characterize some sort of energy density associated with quantum fluctuations of space-time or an energy scale associated with the masses of the devices used in the measurement process.]

Since  $\alpha \geq 1$  in all Quantum-Gravity approaches believed to support deformed dispersion relations, and since it is quite plausible that  $E_{QG}$  would be rather close to  $10^{19} \text{GeV}$ , it appears likely that the factor  $(E^*/E_{QG})^{\alpha-1}$  would suppress the random-walk noise effect.

### 3.6 Noise with $f^{-5/6}$ amplitude spectral density

In Subsection 3.2 a bound on the measurability of distances based on the Salecker-Wigner procedure was used as motivation for experimental tests of interferometer noise of random-walk type, with  $f^{-1}$  amplitude spectral density and  $\sqrt{T_{obs}}$  root-mean-square deviation. In this Subsection I shall pursue further the observation that the relevant measurability bound could be derived by simply insisting that the devices do not turn into black holes. That observation allowed to derive Eq. (11), which expresses the minimum uncertainty  $\delta D$  on the measurement of a distance  $D$  (*i.e.* the measurability bound for  $D$ ) as proportional to  $\sqrt{T_{obs}}$  and  $\sqrt{(1/S_b + 1/S_d)}$ . Within that derivation the minimum uncertainty is therefore obtained in correspondence of the minimum value of  $1/S_b + 1/S_d$  consistent with the structure of the measurement procedure. Since, given the size  $S_b$  of the bodies whose distance is being measured, the minimum of  $1/S_b + 1/S_d$  corresponds to  $\max[S_d]$  I was led to consider how large  $S_d$  could be while still allowing to disregard any non-rigidity in the quantum motion of the device (which would otherwise lead to additional contributions to the uncertainties). I managed to motivate the random-walk noise scenario by simply assuming that  $\max[S_d]$  be independent of the accuracy  $\delta D$  that the observer would wish to achieve. However, as already argued earlier in this Article, the same physical intuition that motivates some of the fuzzy space-time scenarios here considered also suggests that Quantum Gravity might require a novel measurement theory, possibly involving a new type of relation between system and measuring apparatus. Based on this intuition, it seems reasonable to contemplate the possibility that  $\max[S_d]$  might actually depend on  $\delta D$ .

It is such a scenario that I want to consider in this Subsection. In particular I want to consider the case  $\max[S_d] \sim \delta D$ , which, besides being simple, has the plausible property

that it allows only small devices if the uncertainty to be achieved is small, while it would allow correspondingly larger devices if the observer was content with a larger uncertainty. This is also consistent with the idea that elements of non-rigidity in the quantum motion of extended devices might be negligible if anyway the measurement is not aiming for great accuracy, while they might even lead to the most significant contributions to the uncertainty if all other sources of uncertainty are very small. Salecker and Wigner [15] would also argue that “large” devices are not suitable for very accurate space-time measurements (they end up being “in the way” of the measurement procedure) while they might be admissible if space-time is being probed rather softly.

In this scenario with  $\max[S_d] \sim \delta D$ , Eq. (11) takes the form

$$\delta D \geq \sqrt{\left(\frac{1}{S_b} + \frac{1}{S_d}\right) \frac{L_{planck}^2 c T_{obs}}{2}} \geq \sqrt{\frac{L_{planck}^2 c T_{obs}}{2 \delta D}}, \quad (21)$$

which actually gives

$$\delta D \geq \left(\frac{1}{2} L_{planck}^2 c T_{obs}\right)^{1/3}. \quad (22)$$

As already done with the other measurability bounds discussed in this Article, I shall take Eq. (22) as motivation for the investigation of the corresponding fuzziness scenarios characterised by

$$\sigma_D \sim \left(\tilde{L}_{QG}^2 c T_{obs}\right)^{1/3}. \quad (23)$$

Notice that in this equation I replaced  $L_{planck}$  with a generic length scale  $\tilde{L}_{QG}$ , since it is possible that the heuristic argument leading to Eq. (23) might have captured the qualitative structure of the phenomenon while providing an incorrect estimate of the relevant length scale. As discussed later in this Article significant bounds on this length scale can be set by experimental data, so we can take a phenomenological attitude toward  $\tilde{L}_{QG}$ .

As one can verify for example using Eq. (5), the  $T_{obs}^{1/3}$  dependence of  $\sigma_D$  is associated with displacement amplitude spectral density with  $f^{-5/6}$  behaviour:

$$\mathcal{S}(f) = f^{-5/6} (\tilde{L}_{QG}^2 c)^{1/3}. \quad (24)$$

For  $\tilde{L}_{QG} \sim 10^{-35} m$  this equation would predict  $\mathcal{S}(f) = f^{-5/6} \cdot (3 \cdot 10^{-21} m Hz^{1/3})$ .

## 4 COMPARISON WITH GRAVITY-WAVE INTERFEROMETER DATA

From the point of view of the operative definition of fuzzy distance given in Section 2 the scenarios for space-time fuzziness considered in the previous Section can all be characterized in terms of three alternative possibilities for the root-mean-square deviation  $\sigma_D$  associated to the fluctuations induced on  $D$  by conjectured quantum properties of space-time. For convenience I report here the three alternatives for  $\sigma_D$  that I ended up considering:

$$\sigma_D \sim L_{min}, \quad (25)$$

$$\sigma_D \sim \sqrt{L_{QG} c T_{obs}} , \quad (26)$$

$$\sigma_D \sim \left( \tilde{L}_{QG}^2 c T_{obs} \right)^{1/3} . \quad (27)$$

The discussion of the fuzziness scenarios considered in the previous Section was consistent with the assumption that the length scale characterizing fuzziness (be it  $L_{min}$ ,  $L_{QG}$  or  $\tilde{L}_{QG}$ ) would be a general fundamental property of Quantum Gravity, independent of the peculiarities of the specific experimental setup and of its environment. However, the fuzziness scenario considered in Subsection 3.5 provided some motivation for the idea that at least  $L_{QG}$  (if Eq. (26) was to be realized in the physical world) might not be a universal length scale, *i.e.* it might depend on some specific properties of the experimental setup and in particular in some contexts (those with small  $E^*/E_{QG}$ ) one might find  $\tilde{L}_{QG}$  to be significantly smaller than  $L_{planck}$ . The possibility that the “magnitude” of space-time fuzziness might depend on the specific context and experimental setup is also consistent with the arguments which support the possibility of a novel Quantum-Gravity relationship between system and measuring apparatus. If the length scale characterizing fuzziness depended on this relationship it might take different values in different experimental setups.

Setting aside these possible complications associated to a novel Quantum-Gravity relationship between system and measuring apparatus, I shall proceed discussing the bounds set on the length scales  $L_{min}$ ,  $L_{QG}$  or  $\tilde{L}_{QG}$  by available experimental data. Let me start observing that, while conceptually they represent drastic departures from conventional physics, phenomenologically the proposals (25), (26) and (27) appear to encode only minute effects. For example, assuming that  $L_{min}$ ,  $L_{QG}$  and  $\tilde{L}_{QG}$  are not much larger than the Planck length, all of these proposals encode submeter uncertainties on the size of the whole observable universe (about  $10^{10}$  light years). However, the precision [20] of modern gravity-wave interferometers is such that they can provide significant information at least on the proposals (26) and (27). In fact, as already mentioned in Section 2, the operation of gravity-wave interferometers is based on the detection of minute changes in the positions of some test masses (relative to the position of a beam splitter). If these positions were affected by quantum fluctuations of the type discussed above the operation of gravity-wave interferometers would effectively involve an additional source of noise due to Quantum-Gravity. This observation allows to set interesting bounds already using existing noise-level data obtained at the *Caltech 40-meter interferometer*, which has achieved [21] displacement noise levels with amplitude spectral density lower than  $10^{-18} m/\sqrt{Hz}$  for frequencies between 200 and 2000 *Hz*. While these sensitivity levels are still very far from the levels required in order to test proposal (25) (from the analysis reported in Subsection 3.1 it follows that for  $L_{min} \sim L_{planck}$  and  $f \sim 1000 Hz$  the Quantum-Gravity noise induced in that scenario is only of order  $10^{-36} m/\sqrt{Hz}$ ), as seen by straightforward comparison with Eq. (9) these sensitivity levels clearly rule out all values of  $L_{QG}$  down to the Planck length. Actually, even values of  $L_{QG}$  significantly smaller than the Planck length are inconsistent with the data reported in Ref. [21]; in particular, by confronting Eq. (9) with the observed noise level of  $3 \cdot 10^{-19} m/\sqrt{Hz}$  near 450 *Hz*, which is the best achieved at the *Caltech 40-meter interferometer*, one obtains the bound  $L_{QG} \leq 10^{-40} m$ .

While, as mentioned, at present we should allow for some relatively small factor to intervene in the relation between  $L_{QG}$  and  $L_{planck}$ , the exclusion of all values of  $L_{QG}$  down to  $10^{-40} m$  appears to be quite significant, perhaps even problematic, for the proposal (26). In particular, this experimental bound rules out the possibility that (26) might be the result of space-time fluctuations of the random-walk type discussed in Subsection 3.3, with a fluctuation of magnitude  $L_{planck} \sim 10^{-35} m$  for each time interval  $t_{planck} \sim 10^{-44} s$ ; in fact, as shown above, such a picture would lead to values of  $L_{QG}$  not significantly smaller than



$L_{\text{planck}}$ . The fact that this picture is ruled out is perhaps the most striking lesson coming out of available interferometer data. Only a few years ago it might have seemed impossible to test a scenario involving fluctuations of magnitude  $L_{\text{planck}}$ , even if such fluctuations might have been quite frequent (one each  $t_{\text{planck}}$ ). From the point of view of modeling quantum fluctuations of space-time our simple-minded random-walk model might still be useful, but clearly some new element must be introduced in order to temper the associated fuzziness of space-time; for example, as mentioned in closing Subsection 3.3, one might consider the possibility that fluctuations of magnitude  $L_{\text{planck}}$  would not be as frequent as  $1/t_{\text{planck}}$ .

In any case, of course, even more stringent bounds on  $L_{QG}$  are within reach of the next LIGO/VIRGO [22, 23] generation of gravity-wave interferometers. It would seem that very little room for adjustments of the random-walk noise scenario would remain available if also LIGO and VIRGO give negative results for what concerns this scenario.

The sensitivity achieved at the *Caltech 40-meter interferometer* also sets a bound on the proposal (23)-(24). By observing that Eq. (24) would imply Quantum-Gravity noise levels for gravity-wave interferometers of order  $\tilde{L}_{QG}^{2/3} \cdot (10 \text{ m}^{1/3}/\sqrt{Hz})$  at frequencies of a few hundred  $Hz$ , one obtains from the data reported in Ref. [21] that  $\tilde{L}_{QG} \leq 10^{-29} \text{ m}$ . This bound is remarkably stringent in absolute terms, but is still quite far from the range of values one ordinarily considers as likely candidates for length scales appearing in Quantum Gravity. A more significant bound on  $\tilde{L}_{QG}$  should be obtained by the LIGO/VIRGO generation of gravity-wave interferometers. For example, it is plausible [22] that the “advanced phase” of LIGO achieve a displacement noise spectrum of less than  $10^{-20} \text{ m}/\sqrt{Hz}$  near  $100 \text{ Hz}$  and this would probe values of  $\tilde{L}_{QG}$  as small as  $10^{-34} \text{ m}$ .

Looking beyond the LIGO/VIRGO generation of gravity-wave interferometers, one can envisage still quite sizeable margins for improvement by optimizing the performance of the interferometers at low frequencies, where both (9) and (24) become more significant. It appears natural to perform such studies in the quiet environment of space, perhaps through future refinements of LISA-type setups [24].

The indication of the low-frequency range as most promising for Quantum Gravity tests at interferometers should be seen as the most robust result obtained in this Article. The arguments advocated in the previous Section 3 were all rather speculative and it would not be surprising if some of the details of the estimates turned out to be completely off the mark, but the fact that nearly all of those arguments pointed us toward the low-frequency region might nevertheless be indicative. I hope that, in spite of the heuristic nature of the arguments advocated in the previous Section, colleagues on the experimental side will take the low-frequency hint into consideration in planning future experimental tests of quantum properties of space-time.

## 5 ABSOLUTE MEASURABILITY BOUND FOR THE AMPLITUDE OF A GRAVITY WAVE

Up to this point I have discussed how certain plausible quantum properties of space-time would affect the noise levels in interferometers. The fact that I considered the sensitivity levels of gravity-wave interferometers was due to the fact that these are the most advanced modern interferometers, and it was not in any way related to their function as gravity-wave detectors. In this Section 5 I instead consider an aspect of the physics of gravity waves; specifically, I discuss the way in which the interplay between Gravity and Quantum Mechanics could affect the measurability of the amplitude of a gravity wave. The reader should notice that in this Section nothing is assumed of Quantum Gravity: I just combine known properties of Gravity and Quantum Mechanics. This is also different from the analyses

reported in the previous sections which concerned candidate Quantum Gravity phenomena. The motivation for considering those Quantum Gravity phenomena came from combining known properties of Gravity and Quantum Mechanics, but the phenomena (*e.g.* the models for space-time fuzziness) could not be seen as straightforward combination of Gravity and Quantum Mechanics, they truly pertained to a novel type of physics.

Having clarified in which sense this Section represents a deviation from the main bulk of observations reported in the present Article, let me start the discussion by reminding the reader of the fact that, as already mentioned in Section 2, the interference pattern generated by a modern interferometer can be remarkably sensitive to changes in the positions of the mirrors relative to the beam splitter, and is therefore sensitive to gravitational waves (which, as described in the *proper reference frame* [20], have the effect of changing these relative positions). With just a few lines of simple algebra one can show that an ideal gravitational wave of amplitude  $h$  and reduced<sup>12</sup> wavelength  $\lambda_{gw}^o$  propagating along the direction orthogonal to the plane of the interferometer would cause a change in the interference pattern as for a phase shift of magnitude  $\Delta\phi = D_L/\lambda^o$ , where  $\lambda^o$  is the reduced wavelength of the laser beam used in the measurement procedure and [20, 46]

$$D_L \sim 2 h \lambda_{gw}^o \left| \sin \left( \frac{L}{2\lambda_{gw}^o} \right) \right| , \quad (28)$$

is the magnitude of the change caused by the gravitational wave in the length of the arms of the interferometer. (The changes in the lengths of the two arms have opposite sign [20].)

As already mentioned in Section 2, modern techniques allow to construct gravity-wave interferometers with truly remarkable sensitivity; in particular, at least for gravitational waves with  $\lambda_{gw}^o$  of order  $10^3 Km$ , the next LIGO/VIRGO generation of detectors should be sensitive to  $h$  as low as  $3 \cdot 10^{-22}$ . Since  $h \sim 3 \cdot 10^{-22}$  causes a  $D_L$  of order  $10^{-18}m$  in arms lengths  $L$  of order  $3Km$ , it is not surprising that in the analysis of gravity-wave interferometers, in spite of their huge size, one ends up having to take into account [20] the type of quantum effects usually significant only for the study of processes at or below the atomic scale. In particular, there is the so-called *standard quantum limit* on the measurability of  $h$  that results from the combined minimization of *photon shot noise* and *radiation pressure noise*. While a careful discussion of these two noise sources (which the interested reader can find in Ref. [20]) is quite insightful, here I shall rederive this *standard quantum limit* in an alternative<sup>13</sup> and straightforward manner (also discussed in Ref. [46]), which relies on the application of Heisenberg's uncertainty principle to the position and momentum of a mirror relative to the position of the beam splitter. This can be done along the lines of my analysis of the Salecker-Wigner procedure for the measurement of distances. Since the mirrors and the beam splitter are macroscopic, and therefore the corresponding momenta and velocities are related non-relativistically, Heisenberg's uncertainty principle implies that

$$\delta x \delta v \geq \frac{\hbar}{2} \left( \frac{1}{M_m} + \frac{1}{M_b} \right) \geq \frac{\hbar}{2M_m} , \quad (29)$$

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<sup>12</sup>I report these results in terms of reduced wavelengths  $\lambda^o$  (which are related to the wavelengths  $\lambda$  by  $\lambda^o = \lambda/(2\pi)$ ) in order to avoid cumbersome factors of  $\pi$  in some of the formulas.

<sup>13</sup>While the *standard quantum limit* can be equivalently obtained either from the combined minimization of *photon shot noise* and *radiation pressure noise* or from the application of Heisenberg's uncertainty principle to the position and momentum of the mirror, it is this author's opinion that there might actually be a fundamental difference between the two derivations. In fact, it appears (see, *e.g.*, Ref. [32] and references therein) that the limit obtained through combined minimization of *photon shot noise* and *radiation pressure noise* can be violated by careful exploitation of the properties of squeezed light, whereas the limit obtained through the application of Heisenberg's uncertainty principle to the position and momentum of the mirror is so fundamental that it could not possibly be violated.

where  $\delta x$  and  $\delta v$  are the uncertainties in the relative position and relative velocity,  $M_m$  is the mass of the mirror,  $M_b$  is the mass of the beam splitter. [Again, the relative motion is characterised by the *reduced mass*, which is given in this case by  $(1/M_m + 1/M_b)^{-1}$ .]

Clearly, the high precision of the planned measurements requires [20, 46] that the position of the mirrors be kept under control during the whole time  $2L/c$  that the beam spends in between the arms of the detector before superposition. When combined with (29) this leads to the finding that, for any given value of  $M_m$ , the  $D_L$  induced by the gravitational wave can be measured only up to an irreducible uncertainty, the so-called *standard quantum limit*:

$$\delta D_L \geq \delta x + \delta v 2 \frac{L}{c} \geq \delta x + \frac{\hbar L}{c M_m \delta x} \geq \sqrt{\frac{\hbar L}{c M_m}}. \quad (30)$$

Here of course the reader will realize that the conceptual steps are completely analogous to the one of the discussion given in Section 3 of the Salecker-Wigner procedure for the measurement of distances. The similarities between the analysis of measurability for Salecker-Wigner distance measurements and the analysis of measurability by gravity-wave interferometers are a consequence of the fact that in both contexts a light signal is exchanged and the measurement procedure requires that the relative positions of some devices be known with high accuracy during the whole time that the signal spends between the bodies.

The case of gravity-wave measurements is a canonical example of my general argument that the infinite-mass classical-device limit underlying ordinary Quantum Mechanics is inconsistent with the nature of gravitational measurements. As the devices get more and more massive they not only increasingly disturb the gravitational/geometrical observables, but eventually (well before reaching the infinite-mass limit) they also render impossible [9, 11] the completion of the procedure of measurement of gravitational observables. In trying to assess how this observation affects the measurability of the properties of a gravity wave let me start by combining Eqs. (28) and (30):

$$\delta h = \delta \left( \frac{D_L}{L} \right) = h \frac{\delta D_L}{D_L} \geq \frac{\sqrt{\frac{\hbar L}{c M_m}}}{2 \lambda_{gw}^o \left| \sin \left( \frac{L}{2 \lambda_{gw}^o} \right) \right|}. \quad (31)$$

In complete analogy with some of the observations made in Section 3 concerning the measurability of distances, I observe that, when gravitational effects are taken into account, there is an obvious limitation on the mass of the mirror:  $M_m$  must be small enough that the mirror does not turn into a black hole.<sup>14</sup> In order for the mirror not to be a black hole one requires  $M_m < \hbar S_m / (c L_{planck}^2)$ , where  $L_{planck} \sim 10^{-33} cm$  is the Planck length and  $S_m$  is the *size* of the region of space occupied by the mirror. This observation combined with (31) implies that one would have obtained a bound on the measurability of  $h$  if one found a maximum allowed mirror size  $S_m$ . In estimating this maximum  $S_m$  one can be easily led to two extreme assumptions that go in opposite directions. It is perhaps worth commenting on the weaknesses of these assumptions, as this renders more intuitive the discussion of the correct estimate. On one extreme, one could suppose that in order to achieve a sensitivity to  $D_L$  as low as  $10^{-18} m$  it might be necessary to “accurately position” each  $10^{-36} m^2$  surface element of the mirror. If this was really necessary, our line of argument would then lead to a rather large measurability bound. Fortunately, the phase of the wavefront of the reflected light beam is determined by the average position of all the atoms across the beam’s width, and microscopic irregularities in the structure of the mirror only lead to scattering of a small fraction of light out of the beam. This suggests that in our analysis the size of the mirror

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<sup>14</sup>This is of course a very conservative bound, since a mirror stops being useful as a device well before it turns into a black hole, but even this conservative approach leads to an interesting finding.

should be assumed to be of the order of the width of the beam [20]. Once this is taken into account another extreme assumption might appear to be viable. In fact, especially when guided by intuition coming from table-top interferometers, one might simply assume that the mirror could be *attached* to a very massive body. Within this assumption our line of argument would not lead to any bound on the measurability of  $h$ . However, whereas for the type of accuracies typically involved in table-top experiments the idealization of a mirror *attached* to the table is appropriate<sup>15</sup>, in gravity-wave interferometers the precision is so high that it becomes necessary to take into account the fact that no *attachment procedure* can violate Heisenberg's Uncertainty Principle (and causality). Clearly by *attaching* a mirror of size  $S_m$  to a massive body one would not avoid the minimum uncertainty  $\sqrt{cT L_{\text{planck}}^2/S_m}$  in the position of the mirror over a time  $T$ . Actually, mirrors provide a good context in which to illustrate the interplay between gravitational and quantum properties of devices. If a mirror is extended enough that one might not be able to neglect the fact it is not really moving rigidly with its center of mass, additional contributions to quantum uncertainties are found. The relative motion of different parts of the mirror is, of course, not "immune" to the Uncertainty Principle, and over a time  $T_{\text{obs}}$  the relative position of different parts of the mirror will necessarily have an uncertainty proportional to  $\sqrt{\hbar T_{\text{obs}}/m}$ , where  $m$  is the mass of the small portions of the mirror whose relative position we are considering (rather than the larger mass of the entire mirror). In order to be able to use the mirror these uncertainties must be small enough to render the mirror consistent with the level of accuracy required by the measurement.<sup>16</sup> These considerations further supports the point that in the present analysis the size  $S_m$  of the mirror should be taken to be of the order of the width of the beam, rather than being replaced by the size of some massive body *attached* to the mirror. In light of these considerations one clearly sees an upper bound on  $S_m$ ; in fact, if the width of the beam (and therefore the effective size of the mirror) is larger than the  $\lambda_{\text{gw}}^o$  of the gravity wave<sup>17</sup> which one is planning to observe, that same gravity wave would cause phenomena that would not allow the proper completion of the measurement procedure (*e.g.* deforming the mirror and leading to a nonlinear relation between  $D_L$  and  $h$ ). One concludes that  $M_m$  should be smaller than  $\hbar\lambda_{\text{gw}}^o/(cL_{\text{planck}}^2)$ , and this can be combined with (31) to obtain the measurability bound

$$\delta h > \frac{L_{\text{planck}}}{2\lambda_{\text{gw}}^o} \frac{\sqrt{L/\lambda_{\text{gw}}^o}}{\left| \sin\left(\frac{L}{2\lambda_{\text{gw}}^o}\right) \right|}. \quad (32)$$

This result not only sets a lower bound on the measurability of  $h$  with given arm's length  $L$ , but also encodes an absolute (*i.e.* irrespective of the value of  $L$ ) lower bound, as a result of the

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<sup>15</sup>Of course, in a table-top experiment it is possible to *attach* a mirror to the table in such a way that the noise associated to the residual relative motion of the mirror with respect to the table be smaller than all other sources of noise.

<sup>16</sup>It is worth emphasizing that ideal mirrors (like other ideal classical devices) are consistent with the laws of ordinary (non-gravitational) Quantum Mechanics, but are inadmissible once gravitational interactions are turned on. In the limit of ordinary Quantum Mechanics in which each small portion of the mirror has infinite mass the *Uncertainty Principle* ceases to induce non-rigidity in the mirror. However, when gravitational interactions combine with Quantum Mechanics this infinite mass limit is inconsistent with the procedure of measurement of gravitational observables.

<sup>17</sup>Note that for the gravitational waves to which LIGO will be most sensitive, which have  $\lambda_{\text{gw}}^o$  of order  $10^3 \text{Km}$ , the requirement  $S_m < \lambda_{\text{gw}}^o$  simply states that the size of mirrors should be smaller than  $10^3 \text{Km}$ . This bound might appear very conservative, but I am trying to establish an *in principle* limitation on the measurability of  $h$ , and therefore I should not take into account that present-day technology is very far from being able to produce a  $10^3 \text{Km}$  mirror with the required profile precision.

fact that the function  $\sqrt{x}/|\sin(x/2)|$  has an absolute minimum:  $\min[\sqrt{x}/\sin(x/2)] \sim 1.66$ . This novel measurability bound is a significant departure from the principles of ordinary Quantum Mechanics, especially in light of the fact that it describes a limitation on the measurability of a single observable (the amplitude  $h$  of a gravity wave), and that this limitation turns out to depend on the value (not the associated uncertainty) of another observable (the reduced wavelength  $\lambda_{gw}^o$  of the same gravity wave). It is also significant that this new bound (32) encodes an aspect of a novel type of interplay between system and measuring apparatus in Quantum-Gravity regimes; in fact, in deriving (32) a crucial role was played by the fact that in accurate measurements of gravitational/geometrical observables it is no longer possible [11] to advocate an idealized description of the devices.

Also the  $T_{obs}$ -dependent bound on the measurability of distances which I reviewed in Section 3 encodes a departure from ordinary Quantum Mechanics and a novel type of interplay between system and measuring apparatus, but the bound (32) on the measurability of the amplitude of a gravity wave (which is one of the new results reported in the present Article) should provide even stronger motivation for the search of formalism in which Quantum Gravity is based on a new mechanics, not exactly given by ordinary Quantum Mechanics. In fact, while one might still hope to find alternatives to the Salecker-Wigner measurement procedure that allow to measure distances evading the bound (8) (or its  $\delta D \sim \max[S_d]$  version (23)), it appears hard to imagine that there could be anything (even among “gedanken laboratories”) better than an interferometer for measurements of the amplitude of a gravity wave.

The fact that in the limit  $\lambda_{gw}^o \rightarrow \infty$  (the no-gravity-wave limit) the bound (32) reduces to the bound  $\delta h > \delta L/L$  is of course consistent with the fact that when no gravity wave is going through the interferometer the only Quantum-Gravity related noise sources (if any) come directly from the distance fuzziness  $\delta L$ , which I considered in the previous Sections. The analysis reported in this Subsection appears to indicate that the interferometer noise associated to distance fuzziness could be simply seen as the  $\lambda_{gw}^o \rightarrow \infty$  limit of a more complicated  $\lambda_{gw}^o$ -dependent type of Quantum-Gravity related noise affecting the observation of gravity waves in a full Quantum Gravity context.

It is also important to realize that the bound (32) cannot be obtained by just assuming that the Planck length  $L_{planck}$  provides the minimum uncertainty for lengths [26]. In fact, if the only limitation was  $\delta D_L \geq L_{planck}$  the resulting uncertainty on  $h$ , which I denote with  $\delta h^{(L_{planck})}$ , would have the property

$$\min[\delta h^{(L_{planck})}] = \min \left[ \frac{L_{planck}}{2 \lambda_{gw}^o \left| \sin \left( \frac{L}{2 \lambda_{gw}^o} \right) \right|} \right] = \frac{L_{planck}}{2 \lambda_{gw}^o}, \quad (33)$$

whereas, exploiting the above-mentioned properties of the function  $\sqrt{x}/|\sin(x/2)|$ , from (32)

one finds<sup>18</sup>

$$\min[\delta h] > \min \left[ \frac{L_{\text{planck}}}{2 \lambda_{gw}^o} \frac{\sqrt{L/\lambda_{gw}^o}}{\left| \sin \left( \frac{L}{2\lambda_{gw}^o} \right) \right|} \right] > \min[\delta h^{(L_{\text{planck}})}] . \quad (34)$$

In general, the dependence of  $\delta h^{(L_{\text{planck}})}$  on  $\lambda_{gw}^o$  is different from the one of  $\delta h$ . Actually, in light of the comparison of (33) with (34) it is amusing to observe that the bound (32) could be seen as the result of a minimum length  $L_{\text{planck}}$  combined with an  $\lambda_{gw}^o$ -dependent correction. This would be consistent with some of the ideas mentioned in Section 3, the energy-dependent effect of *in vacuo* dispersion and the corresponding proposal (19) for distance fuzziness, in which the magnitude of the Quantum Gravity effect depends rather sensitively on some energy-related aspect of the problem under investigation (just like  $\lambda_{gw}^o$  gives the energy of the gravity wave).

It is easy to verify that the bound (32), would not observably affect the operation of even the most sophisticated planned interferometers. However, in the spirit of what I did in the previous Sections considering the operative definition of distances, also for the amplitudes of gravity waves the fact that we have encountered an obstruction in the measurement analysis based on ordinary Quantum Mechanics (and the fact that by mixing Gravity and Quantum Mechanics we have obtained some intuition for novel qualitative features of such gravity-wave amplitudes in Quantum Gravity) could be used as starting point for the proposal of novel Quantum Gravity effects possibly larger than the estimate (32) obtained by naive combination of Gravity and Quantum Mechanics without any attempt at a fully Quantum-Gravity picture of the phenomenon. Although possibly very interesting, these fully Quantum-Gravity scenarios for the properties of gravity-wave amplitudes will not be explored in the present Article.

## 6 RELATIONS WITH OTHER QUANTUM GRAVITY APPROACHES

The general strategy for the search of Quantum Gravity which has led to the arguments reviewed and/or presented in the previous sections is evidently quite different from the strategy adopted in other approaches to the unification of Gravity and Quantum Mechanics. [I shall discuss these differences in greater detail in Section 8.] However, it is becoming increasingly clear (especially in discussions and research papers that were motivated by Refs. [4, 8]) that in spite of these differences some common elements of intuition concerning the interplay of Gravity and Quantum Mechanics are emerging. In this Section I want to emphasize these relationships with some Quantum Gravity approaches and at the same time I want to clarify the differences with respect to other Quantum Gravity approaches.

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<sup>18</sup>I am here (for “pedagogical” purposes) somewhat simplifying the comparison between  $\delta h$  and  $\delta h^{(L_{\text{planck}})}$ . As mentioned, in principle one should take into account both uncertainties inherent in the “system” under observation, which are likely to be characterized exclusively by the Planck-length bound, and uncertainties coming from the “measuring apparatus”, which might easily involve other length (or time) scales besides the Planck length. It would therefore be proper to compare  $\delta h^{(L_{\text{planck}})}$ , which would be the only contribution present in the conventional idealization of “classical devices”, with the sum  $\delta h + \delta h^{(L_{\text{planck}})}$ , which, as appropriate for Quantum Gravity, provides a sum of system-inherent uncertainties plus apparatus-induced uncertainties.

## 6.1 Canonical Quantum Gravity

One of the most popular Quantum Gravity approaches (whose popularity might have been the reason for the diffusion of the possibly misleading name “Quantum Gravity”) is the one in which the ordinary canonical formalism of Quantum Mechanics is applied to (some formulation of) Einstein’s Gravity.

While I must emphasize again [11] that some of the observations reviewed and/or reported in the previous sections strongly suggest that Quantum Gravity should require a new mechanics, not exactly given by ordinary Quantum Mechanics, it is nonetheless encouraging that some of the phenomena considered in the previous sections have also emerged in studies of Canonical Quantum Gravity.

The most direct connection was found in the study reported in Ref. [19], which was motivated by Ref. [4]. In fact, Ref. [19] shows that the popular Canonical/Loop Quantum Gravity [38] admits the phenomenon of deformed dispersion relations, with the deformation going linearly with the Planck length.

Concerning the bounds on the measurability of distances it is probably fair to say that the situation in Canonical/Loop Quantum Gravity is not yet clear because the present formulations do not appear to lead to a compelling candidate “length operator.” This author would like to interpret the problems associated with the length operator as an indication that perhaps something unexpected might actually emerge in Canonical/Loop Quantum Gravity as a length operator, possibly something with properties fitting the intuition emerging from the analyses in Subsections 3.2, 3.3, and 3.6. Actually, the random-walk space-time fuzziness models discussed in Subsection 3.3 might have a (somewhat weak, but intriguing) connection with “Quantum Mechanics applied to Gravity” at least to the level seen by comparison with the scenario discussed in Ref. [47], which was motivated by the intuition that is emerging from investigations of the Canonical/Loop Quantum Gravity. The “moves” of Ref. [47] share many of the properties of the “random steps” of my random-walk models. Unfortunately, in both approaches one is still searching for a more complete description of the dynamics, and particularly for estimates of how frequently (in time) a  $L_{\text{planck}}$ -size step/move is taken.

## 6.2 Non-commutative geometry and deformed symmetries

Although this was not emphasized in the present Article, some of the Quantum Gravity intuition emerging from the observations in the previous sections fits rather naturally within certain approaches based on non-commutative geometry and deformed symmetries. In particular, there is growing evidence [10, 43] that theories living in the non-commutative Minkowski space proposed in Refs. [48, 42], which involves a dimensionful (possibly Planck length related) deformation parameter, would host both the phenomenon of Planck-length-linear deformations of dispersion relations and phenomena setting  $T_{\text{obs}}$ -dependent bounds on the measurability of distances.

In general, the possibility of dimensionful deformations of symmetries [42, 49] might be quite natural [11] if indeed the relation between system and measuring apparatus is modified at the Quantum Gravity level. For example, the symmetries we observe in ordinary Quantum Mechanics experiments at low energies might be the ones valid in the limit in which the interaction between system and measuring apparatus can be neglected. The dimensionful parameter characterizing the deformation of symmetries could mark a clear separation between (high-energy) processes in which the violations of ordinary symmetries are large and (low-energy) processes in which ordinary symmetries hold to a very good approximation.

On the subject of quantum deformations of space-time symmetries interesting work has also been devoted (see, *e.g.*, Refs. [50, 51]) to frameworks that would host a bound on the measurability of distances of type (1).

### 6.3 Critical and non-critical String Theories

Unfortunately, in the popular Quantum Gravity approach based on Critical Superstring Theory<sup>19</sup> not many results have been derived concerning directly the quantum properties of space-time. Perhaps the most noticeable such results are the ones on limitations on the measurability of distances emerged in the scattering analyses reported in Refs. [27, 29], which I already mentioned in Subsection 3.1, since they provide support for the hypothesis that also Critical Superstring Theory might host a bound on the measurability of distances of type (1).

A rather different picture is emerging (within the difficulties of this rich formalism) in Liouville (non-critical) String Theory [17], whose development was partly motivated by intuition concerning the “Quantum Gravity vacuum” that is rather close to the one traditionally associated to the works of Wheeler [12] and Hawking [13]. Evidence has been found [18] in Liouville String Theory supporting the validity of deformed dispersion relations, with the deformation going linearly with the Planck/string length. In the sense clarified in Subsections 3.4 this approach might also host a bound on the measurability of distances which grows with  $\sqrt{T_{obs}}$ .

### 6.4 Other types of measurement analyses

In light of the scarce opportunities to get any experimental input in the search for Quantum Gravity, it is not surprising that many authors have been seeking some intuition by formal analyses of the ways in which the interplay between Gravity and Quantum Mechanics could affect measurement procedures. A large portion of these analyses produced a “ $\min[\delta D]$ ” with  $D$  denoting a distance; however, the same type of notation was used for structures defined in significantly different manner. Also different meanings have been given by different authors to the statement “absolute bound on the measurability of an observable.” Quite important for the topics here discussed are the differences (which might not be totally transparent as a result of this unfortunate choice of overlapping notations) between the approach advocated in the present Article (and in Refs. [8, 9, 11]) and the approaches advocated in Refs. [15, 34, 35, 52]. In the present Article “ $\min[\delta D]$ ” denotes an absolute limitation on the measurability of a distance  $D$ . The studies [15, 34, 52] analyzed the interplay of Gravity and Quantum Mechanics in defining a net of time-like geodesics, and in those studies “ $\min[\delta D]$ ” characterizes the maximum “tightness” achievable for the net of time-like geodesics. Moreover, in Refs. [15, 34, 35, 52] it was required that the measurement procedure should not affect/modify the geometric observable being measured, and “absolute bounds on the measurability” were obtained in this specific sense. Instead, here and in Refs. [9, 11] I allowed the possibility for the observable which is being measured to depend also on the devices (the underlying view is that observables in Quantum Gravity would always be, in a sense, shared properties of “system” and “apparatus”), and I only required that the nature of the devices be consistent with the various stages of the measurement procedure (*e.g.*, a black-hole device would not allow some of the required exchanges of signal). My measurability bounds are therefore to be intended from this more fundamental perspective, and this is crucial for the possibility that these measurability bounds be associated to a fundamental Quantum-Gravity mechanism for “fuzziness” (quantum fluctuations of space-time). The analyses reported in Refs. [15, 34, 35, 52] did not include any reference to fuzzy space-times of the type operatively defined in Section 2.

The more fundamental nature of the bounds I obtained is also crucial for the arguments suggesting [9, 11] that Quantum Gravity might require a new mechanics, not exactly given

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<sup>19</sup>As already mentioned the mechanics of String Theory is just an ordinary Quantum Mechanics, the novelty of the approach comes from the fact that the fundamental dynamical entities are extended objects rather than point particles.



by ordinary Quantum Mechanics. The analyses reported in Refs. [15, 34, 35, 52] did not include any reference to this possibility.

I also notice that the conjectured relation between measurability bounds and noise levels in interferometers (*e.g.* the ones characterized by  $S(f) \sim f^{-1}$  or  $S(f) \sim f^{-5/6}$ ) is based on the dependence of the measurability bounds on the time of observation  $T_{obs}$ . In fact, this  $T_{obs}$ -dependence has been here emphasized, while in Refs. [34, 35, 52] the emphasis was placed on observed lengths rather than on the time needed to observe them.

Having clarified that there is a “double difference” (different “*min*” and different “ $\delta D$ ”) between the meaning of  $\min[\delta D]$  adopted in the present Article and the meaning of  $\min[\delta D]$  adopted in Refs. [15, 34, 35, 52], it is however important to notice that the studies reported in Refs. [34, 35, 52] were among the first studies which showed how in some aspects of measurement analysis the Planck length might appear together with other length scales in the problem. For example, a Quantum Gravity effect naturally involving something of length-squared dimensions might not necessarily go like  $L_{planck}^2$ , in some case it could go like  $\Lambda L_{planck}$ , with  $\Lambda$  some other length scale in the problem. Some of my arguments are examples of this possibility; in particular, I find in some cases relations of the type (see, *e.g.*, Eq. (6))

$$\delta D \geq \delta x^* + \frac{A}{\delta x^*} \geq \sqrt{A}, \quad (35)$$

where  $A$ , which has length-squared dimensions, turns out to be given by the product of the  $L_{planck}$ -like small fundamental length  $L_{QG}$  and the typically larger length scale  $cT_{obs}$ .

Interestingly, the analysis of the interplay of Gravity and Quantum Mechanics in defining a net of time-like geodesics reported in Ref. [34] concluded that the maximum “tightness” achievable for the geodesics would be characterized by  $\sqrt{L_{planck}^2 R^{-1} s}$ , where  $R$  is the radius of the (spherically symmetric) clocks whose world lines define the network of geodesics, and  $s$  is the characteristic distance scale over which one is intending to define such a network. The  $\sqrt{L_{planck}^2 R^{-1} s}$  maximum tightness discussed in Ref. [34] is formally analogous to my Eq. (11), but, as clarified above, this “maximum tightness” was defined in a way that is very (“doubly”) different from my “ $\min[\delta D]$ ”, and therefore the two proposals have completely different physical implications. Actually, in Ref. [34] it was also stated that for a single geodesic distance (which might be closer to the type of distance measurability analysis reported here and in Refs. [9, 11]) one could achieve accuracy significantly better than the formula  $\sqrt{L_{planck}^2 R^{-1} s}$ , which was interpreted in Ref. [34] as a direct result of the structure of a network of geodesics.

Relations of the type  $\min[\delta D] \sim (L_{planck}^2 D)^{(1/3)}$ , which are formally analogous to Eq. (22), were encountered in the analysis of maximum tightness achievable for a geodesics network reported in Ref. [52] and in the analysis of measurability of distances reported in Ref. [35]. Although once again the definitions of “*min*” and “ $\delta D$ ” used in these studies are completely different from the ones relevant for the “ $\min[\delta D]$ ” of Eq. (22), the analyses reported in Ref. [35, 52] do provide some additional motivation for the scenario (22), at least in as much as they give examples of the fact that behaviour of the type  $L_{planck}^{2/3}$  can naturally emerge in Quantum-Gravity measurement analyses.

## 6.5 Other interferometry-based Quantum-Gravity studies

Several authors have put forward ideas which combine in one or another way some aspects of interferometry and candidate Quantum Gravity phenomena. While the viewpoints and the results of all of these works are significantly different from the ones of the present Article, it

seems appropriate to at least mention briefly these studies, for the benefit of the interested reader.

A first example, on which I shall return in the next Section, is provided by the idea [3] that we might be able to use modern gravity-wave interferometers to investigate certain candidate early-universe String Theory effects.

The studies reported in Ref. [53] (and references therein) have considered how certain effectively stochastic properties of space-time would affect the evolution of quantum-mechanical states. The stochastic properties there considered are different from the ones discussed in the present Article, but were introduced within a similar viewpoint, *i.e.* stochastic processes as effective description of quantum space-time processes. The implications of these stochastic properties for the evolution of quantum-mechanical states were modeled via the formalism of “primary state diffusion”, but only rather crude models turned out to be treatable. Atom interferometers were found to have properties suitable for tests of this scenario. I should however emphasize that in Ref. [53] the proposed tests concerned the Quantum Mechanics of systems leaving in a fuzzy space-time, whereas here and in Ref. [8] I have discussed direct tests of effectively stochastic properties of space-time.

The studies reported in Refs. [32, 33] are more closely related to the physics of gravity-wave interferometers. In particular, combining a detailed analysis of certain aspects of interferometry and the assumption that quantum space-time effects could be estimated using ordinary Quantum Mechanics applied to Einstein’s gravity, Refs. [32, 33] developed a model of Quantum-Gravity induced noise for interferometers which fits within the scenario I here discussed in Subsection 3.1. [Actually, Refs. [32, 33] discuss in greater detail the spectral features encoded in Eqs. (3)-(4), while, as explained in Subsection 3.1, it was for me sufficient to provide a simplified discussion.] As mentioned in Subsection 3.1, it is not surprising that the assumption that Quantum Gravity be given by an ordinary Quantum Mechanics applied to (some formulation of) Einstein’s gravity would lead to noise levels of the type encoded in Eqs. (3)-(4).

The recent paper Ref. [54] proposed certain quantum properties of gravity waves and discussed the implications for gravity-wave interferometry. Let me emphasize that instead the effects considered here and in Ref. [8] concern the properties of the interferometer and would affect the operation of any interferometer whether or not it would be used to detect gravity waves. Here and in Ref. [8] the emphasis on modern gravity-wave interferometers is only due to the fact that these interferometers, because of the extraordinary challenges posed by the detection of classical gravity waves, are the most advanced interferometers available and therefore provide the best opportunity to test scenarios for Quantum-Gravity induced noise in interferometers.

## 7 A QUANTUM-GRAVITY PHENOMENOLOGY PROGRAMME

While opportunities to test experimentally the nature of the interplay between Gravity and Quantum Mechanics remain extremely rare, the proposals now available [2, 3, 4, 5, 6, 8] represent a small fortune with respect to the expectations of not many years ago. We have finally at least reached the point that the most optimistic/speculative estimates of Quantum Gravity effects can be falsified. In searching for even more opportunities to test Quantum Gravity it is useful to analyze the proposals put forward in Refs. [2, 3, 4, 5, 6, 8] as representatives of the two generic mechanisms that one might imagine to use in Quantum-Gravity experiments. Let me comment here on these mechanisms. The most natural discovery strategy would of course resort to strong Quantum Gravity effects, of the type we expect for collisions of elementary particles endowed with momenta of order the Planck mass ( $10^{19} GeV$ ). Since presently and for the foreseeable future we do not expect to

be able to set up such collisions, the only opportunities to find evidence of strong Quantum Gravity effects should be found in natural phenomena (*e.g.* astrophysical contexts that might excite strong Quantum Gravity effects) rather than in controlled laboratory setups. An example is provided by the experiment proposed in Ref. [3] which would be looking for residual traces of some strong Quantum Gravity effects<sup>20</sup> (specifically, Critical Superstring Theory effects[55]) which might have occurred in the early Universe.

Another class of Quantum Gravity experiments is based on physical contexts in which small Quantum Gravity effects lead to observably large signatures thanks to the interplay with a naturally large number present in such contexts. This is the basic mechanism underlying all the proposals in Refs. [2, 4, 5, 6] and underlying the interferometric studies of space-time fuzziness proposed in Ref. [8] which I have here discussed in detail. For the interferometric studies which I am proposing the large number is essentially provided by the ratio between the inverse of the Planck time and the typical frequencies of operation of gravity-wave interferometers. In practice if some of the space-time fuzziness scenarios discussed in Section 3 capture actual features of quantum space-time, in a time as long as the inverse of the typical gravity-wave interferometer frequency of operation an extremely large number of minute quantum fluctuations in the distance  $D$  could add up. A large sum of small quantities can give a sizeable final result, and in fact this final result would be observable if for example the noise induced by fuzziness was characterised by  $f^{-2}cL_{QG}$  which is comparable in size to the corresponding quantity  $[S_{exp}(f)]^2$  characterising the noise levels achievable with modern interferometers.

For the physical context of gamma rays reaching us from far away astrophysical objects the large number can be provided by the ratio between the time travelled by the gamma rays and the time scale over which the signal presents significant structure (time spread of peaks etc.). The proposal made in Ref. [4] basically uses the fact that this allows to add up a very large number of very minute dispersion-inducing Quantum Gravity effects, and if the deformation of the dispersion relation goes linearly with the Planck length the resulting energy-dependent time-delay turns out to be comparable to the time scale that characterizes some of these astrophysical signals, thereby allowing a direct test of the Quantum Gravity scenario.

Similarly, experiments investigating the quantum phases induced by large gravitational fields [5, 6, 7] (the only aspect of the interplay between Gravity and Quantum Mechanics on which we already have positive “discovery” data [6, 7]) exploit the fact that gravitational forces are additive and therefore, for example, gravitational effects due to the earth are the result of a very large number of very minute gravitational effects (instead we would not be able to measure the quantum phases induced by a single elementary particle).

The large number involved in the possibility that Quantum Gravity effects might leave an observable trace in some aspects of the phenomenology of the neutral-kaon system [2] cannot be directly interpreted as a the number of minute Quantum Gravity effects to which the system is exposed. It is rather that the conjectured Quantum Gravity effects would involve in addition to the small dimensionless ratio between the energy of the kaons and the Planck energy also a very large dimensionless ratio [2] characterising the physics of neutral kaons.

This idea of figuring out ways to put together many minute effects (which until a short time ago had been strangely dismissed by the Quantum Gravity community) has a time honored tradition in physics. Perhaps the clearest example is the particle-physics experiment setting bounds on proton lifetime. The relevant dimensionless ratio characterising

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<sup>20</sup>Most of the effects considered in Ref. [3] actually concern the interplay between classical Gravity and Quantum Mechanics, so they pertain to a very special regime of Quantum Gravity. This is also true of the experiments on gravitationally induced quantum phases [5, 6, 7]. Instead the experiments discussed here and in Refs. [2, 4, 8] concern proposed quantum properties of space-time itself, and could therefore probe even more deeply the structure of Quantum Gravity.

proton-decay analyses is extremely small (somewhere in the neighborhood of  $10^{-64}$ , since it is given by the fourth power of the ratio between the mass of the proton and the grandunification scale), but by keeping under observation a correspondingly large number of protons experimentalists are managing<sup>21</sup> to set highly significant bounds.

Another point of contact between proposed Quantum Gravity experiments and proton decay experiments is that a crucial role in rendering the experiment viable is the fact that the process under investigation would violate some of the symmetries of ordinary physics. This plays a central role in the experiments proposed in Refs. [2, 4].

## 8 MORE ON A LOW-ENERGY EFFECTIVE THEORY OF QUANTUM GRAVITY

While the primary emphasis has been on direct experimental tests of crude scenarios for space-time fuzziness, part of this Article has been devoted to the discussion (expanding on what was reported in Refs. [9, 11]) of the properties that one could demand of a theory suitable for a first stage of partial unification of Gravity and Quantum Mechanics. This first stage of partial unification would be a low-energy effective theory capturing only some rough features of Quantum Gravity, possibly associated with the structure of the non-trivial “Quantum Gravity vacuum”.

One of the features that appear desirable for an effective low-energy theory of Quantum Gravity is that its mechanics be not exactly given by ordinary Quantum Mechanics. I have reviewed some of the arguments [9, 11] in support of this hypothesis when I discussed the Salecker-Wigner setup for the measurement of distances, and showed that the problems associated with the infinite-mass classical-device limit provide encouragement for the idea that the analysis of Quantum Gravity experiments should be fundamentally different from the one of the experiments described by ordinary Quantum Mechanics. A similar conclusion was already drawn in the context of attempts (see, *e.g.*, Ref. [14]) to generalize to the study of the measurability of gravitational fields the famous Bohr-Rosenfeld analysis [37] of the measurability of the electromagnetic field. In fact, in order to achieve the accuracy allowed by the formalism of ordinary Quantum Mechanics, the Bohr-Rosenfeld measurement procedure resorts to ideal test particles of infinite mass, which would of course not be admissible probes in a gravitational context [14]. Since all of the (extensive) experimental evidence for ordinary Quantum Mechanics comes from experiments in which the behaviour of the devices can be meaningfully approximated as classical, and moreover it is well-understood that the conceptual structure of ordinary Quantum Mechanics makes it only acceptable as the theoretical framework for the description of the outcomes of this specific type of experiments, it seems reasonable to explore the possibility that Quantum Gravity might require a new mechanics, not exactly given by ordinary Quantum Mechanics and probably involving a novel (in a sense, “more democratic”) relationship between “measuring apparatus” and “system”.

Other (related) plausible features of the correct effective low-energy theory of Quantum Gravity are novel bounds on the measurability of distances. This appears to be an inevitable consequence of relinquishing the idealized methods of measurement analysis that rely on the artifacts of the infinite-mass classical-device limit. If indeed one of these novel measurability bounds holds in the physical world, and if indeed the structure of the Quantum-Gravity vacuum is non-trivial and involves space-time fuzziness, it appears also plausible that this two features be related, *i.e.* that the fuzziness of space-time would be ultimately responsible for the measurability bounds. It is this scenario which I have investigated here and in Ref. [8], emphasizing the opportunity for direct tests which is provided by modern interferometers.

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<sup>21</sup>This author’s familiarity [56] with the accomplishments of proton-decay experiments has certainly contributed to the moderate optimism for the outlook of Quantum Gravity phenomenology which is implicit in the present Article.

The intuition emerging from these first investigations of the properties of a low-energy effective Quantum Gravity might or might not turn out to be accurate, but additional work on this first stage of partial unification of Gravity and Quantum Mechanics is anyway well motivated in light of the huge gap between the Planck regime and the physical regimes ordinarily accessed in present-day particle-physics or gravity experiments. Results on a low-energy effective Quantum Gravity might provide a perspective on Quantum Gravity that is complementary with respect to the one emerging from approaches based on proposals for a one-step full unification of Gravity and Quantum Mechanics. On one side of this complementarity there are the attempts to find a low-energy effective Quantum Gravity which are necessarily driven by intuition based on direct extrapolation from known physical regimes; they are therefore rather close to the phenomenological realm but they are confronted by huge difficulties when trying to incorporate the physical intuition within a completely new formalism. On the other side there are the attempts of one-step full unification of Gravity and Quantum Mechanics, which usually start from some intuition concerning the appropriate formalism (*e.g.*, “Canonical/Loop Quantum Gravity” [38] or “Critical Superstring Theory” [28, 39]) but are confronted by huge difficulties when trying to “come down” to the level of phenomenological predictions. These complementary perspectives might meet at the mid-way point leading to new insight in Quantum Gravity physics. One instance in which this mid-way-point meeting has already been successful is provided by the candidate phenomenon of Quantum-Gravity induced deformed dispersion relations, which was proposed within a purely phenomenological analysis [4] of the type needed for the search of a low-energy theory of Quantum Gravity, but was then shown [19] to be consistent with the structure of Canonical/Loop Quantum Gravity.

## 9 OUTLOOK

The panorama of opportunities for Quantum Gravity phenomenology is certainly becoming richer. In this Article I have taken the conservative viewpoint that the length scales parametrizing proposed Quantum Gravity phenomena should be somewhere in the neighborhood of the Planck length, but I have taken the optimistic (although supported by various Quantum Gravity scenarios, including Canonical/Loop Quantum Gravity [38, 19]) viewpoint that there should be Quantum Gravity effects going linearly or quadratically with the Planck length, *i.e.* effects which are penalized only by one or two powers of the Planck length.

An exciting recent development is that results in the general area of String Theory have motivated work (see, *e.g.*, Ref. [57]) on theories with large extra dimensions in which rather naturally Quantum Gravity effects would become significant at scales much larger than the conventional Planck length. In such scenarios one expects to find phenomena for which the length scale characterizing the onset of large Quantum-Gravity corrections is much larger than the conventional Planck length.

The example of advanced modern interferometers here emphasized provides further evidence (in addition to the one emerging from Refs. [2, 4]) of the fact that we should eventually be able to find signatures of Quantum Gravity effects if they are linear in the conventional Planck length. If the physical world only hosts effects that are quadratic in the deformation length scale, values of this length scale of order the Planck length would probably be out of reach for the foreseeable future, but effects quadratic in the larger length scales characterizing scenarios of the type in Ref. [57] might be experimentally accessible.

On the theory side an exciting opportunity for future research appears to be provided by the possibility of exchanges of ideas between the more phenomenological/intuitive studies appropriate for the search of a low-energy effective Quantum Gravity and the more rigorous/formal studies used in searches of fully consistent Quantum Gravity theories. As mentioned at the end of the preceding Section, the first example of such an exchange has led

to the exciting realization that deformed dispersion relations linear in the Plack length appear plausible [4, 11, 18] both from the point of view of heuristic phenomenological analyses and are also a rather general prediction of Canonical/Loop Quantum Gravity [19]. Additional exchanges of this type appear likely. For example, the intuition coming from the low-energy effective Quantum Gravity viewpoint on distance fuzziness which I discussed here might prove useful for those Quantum Gravity approaches (again an example is provided by Canonical/Loop Quantum Gravity) in which there is substantial evidence of space-time fuzziness but one has not yet achieved a satisfactory description of fuzzy distances.

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